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Active thermal convection in vibrofluidized granular systems

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Abstract. We present a granular-hydrodynamic model that captures the essence of convection in a fully vibrofluidized granular system. The steady temperature distribution is solved analytically. Numerical simulation shows that the convection always develops through a supercritical bifurcation, with its energy about 5% of the random (heat) one. A comparison calculation is performed for a normal fluid. The convection roll, or an active roll as we call it, has an angular velocity gradient from its interior to exterior. We conclude that active rolls are universal.

PACS. 45.70.Mg Granular flow: mixing, segregation and stratification – 47.20.Bp Buoyancy-driven instability – 47.27.Te Convection and heat transfer

In the study of fluid and vibrofluidized granular systems, convection has attracted particular interest, because convection can not only occur alone, but also simultaneously with (or within) other phenomena (e.g. surface waves or heaps in granular systems), and might even be the source for wave and heap formation. It is well known that in a normal fluid in a gravitational field, free convection can occur if the externally applied vertical temperature gradient is directed downwards and its magnitude exceeds a certain value. The mechanism for this convection is buoyancy due to thermal expansion. In vibrofluidized granular systems, some physical mechanisms for convection have been proposed. One is the friction of the grains with the walls of the container for weakly excited systems [1], and another is the buoyancy (or heat)-driven mechanism for fully fluidized systems. Theoretical techniques utilized to explain convection have mainly been the continuum hydrodynamic theory [2] and large scale molecular dynamics simulations [3]. Owing to the similarity of convection in fully fluidized granular systems and normal fluids, in this paper we model the vibrated granular system as a "thermal" granular fluid. Every grain is treated as a "molecule" of granular fluid, they take part simultaneously in the macroscopic flow motion (if there is convection) as well as the microscopic random or "thermal" motion. The vibrated plate (as an energy source) supplies energy from the bottom to the granular system through grain-grain collisions. Because the collision between grains is inelastic, the kinetic energy of the grains is partially lost and transferred to true heat energy of the system as it spreads from the bottom to the top surface, and thus establishes a gradient of random velocity (or "temperature") from bottom to top surface of the system. As the input energy reaches a critical value, similar to a normal fluid, convection will occur in granular system.

Mathematically we model the vibrofluidized granular system by a set of equations, the momentum equation, the "heat" transfer equation, and the continuity equation, similar to that describing the free convection in a normal fluid [4]. In doing so we suppose that the granular system is excited at high frequency (>30 Hz) and small amplitude (a few particle diameters), therefore, the mean separation between grains, s, and the diameter of grain, d, satisfy $s \ll d$, and the granular system is assumed incompressible. We denote the macroscopic flow velocity by $\mathbf{u}$, and the mean random (or "thermal") velocity by $\mathbf{v}$. The momentum equation is analogous to the Navier Stokes equation, and takes the form

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \bigtriangledown)\mathbf{u} = - \bigtriangledown \frac{p}{\rho} + \nu \bigtriangledown^2 \mathbf{u} - \beta \mathbf{g} T, \qquad (1)$$

where p is the pressure fluctuation, ρ the mean mass density of the granular layer, and $\mathbf{g}$ the acceleration due to gravity. ν is named kinematic viscosity coefficient, and β the thermal-expansion coefficient, $T = mv^2/2$ (m is the mass of the grain) represents the temperature of the system. The heat transfer equation is

$$\frac{\partial T}{\partial t} + \mathbf{u} \cdot \bigtriangledown T = \bigtriangledown \cdot (\chi \bigtriangledown T) + \frac{\nu}{2} \frac{\partial u_i}{\partial x_k} \left(\frac{\partial u_i}{\partial x_k} + \frac{\partial u_k}{\partial x_i} \right) - I, \qquad (2)$$

where χ is the "thermometric" diffusivity, and I the rate of energy dissipation due to the inelasticity of grain-grain collisions. In order to relate the pressure p, the energy dissipation I, and the coefficients ν, β and χ to variable v, it is needed to use some kinetic theory for the granular material, as in reference [5], but here we make use

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of the simple cell model [6], and obtain $p = td\rho v^2/s$, $I = \gamma m v^3/s \equiv \gamma_1 v^3$, $\nu = qd^2 v/s \equiv q_1 v$, $\beta = 6t\rho/mp_0$, and $\chi = rd^2 v/s \equiv r_1 v$, in which $\gamma_1 \equiv \gamma m/s$, $q_1 \equiv qd^2/s$ and $r_1 \equiv rd^2/s$, respectively, p_0 is the averaged pressure, and t, γ, q and r are the dimensionless constants. We will see that the coefficients ν and χ play a more active role here than they do in normal hydrodynamics. They depend upon v. Therefore, equation (1) and equation (2) are coupled not only through the term $\mathbf{u}$ and T, but also through these coefficients. The continuity equation is

$$\bigtriangledown \cdot \mathbf{u} = 0. \tag{3}$$

The dimensionless form of equations (1–3) is obtained in such a way that the magnitude of gravity acceleration, $\mathbf{g}$, the thickness of the granular layer, h, and the mass of the grain, m, are unities, respectively. In a vibrofluidized granular system in a gravitational field, the temperature gradient is related not only to the boundary condition but also to the inelastic collision between grains. Therefore we need to know how this gradient distributes in space, and under what temperature gradient convection will occur. For this purpose, and for simplicity, we solve first the one-dimensional problem of steady state with no macroscopic flow (or convection) in a gravitational field. In dimensionless form, the heat transfer equation (2) is reduced to [7]

$$\frac{d^2 v}{dz^2} + \frac{1}{z}\frac{dv}{dz} - v = 0, \tag{4}$$

where $z = (1-y)/\lambda$, in which $\lambda = (r/\gamma)^{1/2} d$. The boundary condition at the top surface ($y = 1$) is taken as

$$\chi \frac{d}{dy} v^2|_{y=1} = 0. \tag{5}$$

This means that the energy flux vanishes at the top surface. As to the boundary condition at the bottom of the granular layer ($y = 0$), we consider it as follows: In our previous work [8], we used a model of a single sphere colliding completely inelastically with a massive sinusoidally oscillating plate to describe the motion of a layer of vibrated granular material, and obtained the mean input power, P, by the plate for each grain as a function of acceleration of the plate, $\mathit{\Gamma}$. Therefore we take the input power P as the boundary condition at the boundary $y = 0$, i.e.,

$$\chi \frac{d}{dy}\left(\frac{1}{2} v^2\right)|_{y=0} = P. \tag{6}$$

Because the energy flux at the top surface is zero, the energy flux P must be balanced by the energy dissipation due to the inelasticity of the grain-grain collision, and is responsible for the random and convection motion of the grains. The solution to equation (4) is a modified Bessel function of zero order, $I_0(z)$ and $K_0(z)$.

$$v = AI_0(z) + BK_0(z). \tag{7}$$

Here $y \to 1$ corresponds to $z \to 0$. As the function $K_0(z)$ is singular here, B must be set to zero, and v is given by $I_0(z)$ alone, i.e.

$$v = AI_0(z). \tag{8}$$

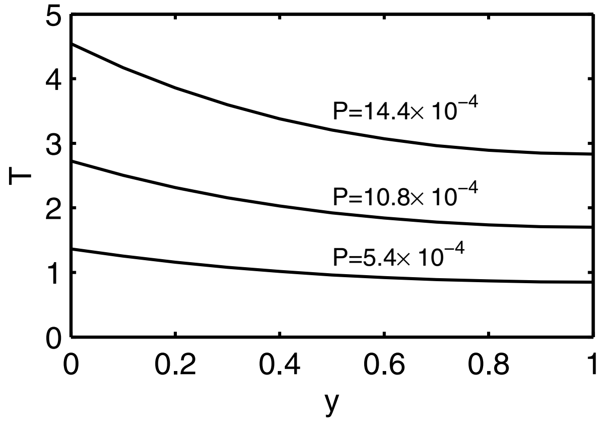

Fig. 1. The analytical static temperature of the linear problem for three values of P.

Inserting this into the boundary condition equation (6), we have

$$A = \left[P/(\chi I_0 \frac{dI_0}{dy})|_{y=0}\right]^{1/2}. \tag{9}$$

Figure 1 is the distribution of the steady static temperature according to the solution equation (8) for three different values of P, which is in qualitative agreement with experiment [9,10].

In principle, in terms of the stability analysis we can obtain the conditions under which convection will occur as in reference [4]. This work is under way. Here we investigate the criteria for convection to occur by solving the equations (1–3) numerically [11]. We shall restrict ourselves to a two-dimensional box with lateral dimension 2, and height 1. The gravity acceleration is in negative y direction, all walls are assumed rigid and free-slip, or the normal components of the velocity at all boundaries are zero. The energy with mean power P inputs into the system from the bottom of the box, the three other walls are assumed to be adiabatic, i.e. the normal heat flux at these walls is zero. The initial conditions are taken as follows: $\mathbf{u}|_{t=0} = 0$, $T|_{t=0} = 0.003$. We limit γ_1, q_1, β and r_1 to be order of 0.1, and vary the input power P. As P is below a critical value, $P_c \sim 0.002$ ($P = 0$ corresponds to $\mathit{\Gamma} = 1$, therefore P_c corresponds to a critical driving acceleration $\mathit{\Gamma}_c > 1$), no convection appears. This can be checked by calculating the macroscopic flow kinetic energy of the system (or convection energy), $E_c = \sum \frac{1}{2} u^2$, summing over all finite-difference grid cells. The reason why we can do this is that since $\bigtriangledown \cdot \mathbf{u} = 0$, so if $E_c = 0$, then circulation $\oint_L \mathbf{u} \cdot d\mathbf{l} = 0$, and vice versa. In this way, we can avoid the choice of the path for calculating the circulation $\oint_L \mathbf{u} \cdot d\mathbf{l}$, since often the number, the position and the orientation of the rolls vary as the input power is increased. In all cases of $P < P_c$, $E_c \sim 0$. The temperature distribution is similar to analytical results (as in Fig. 1) in y direction, and is uniform in x direction. As P is increased to and beyond P_c, the steady state becomes unstable, $E_c > 0$, this means convection occurs. Figure 2 shows the temperature surface and convection patterns for $P = 0.02$. Comparing with Figure 1, we can see that when the convection

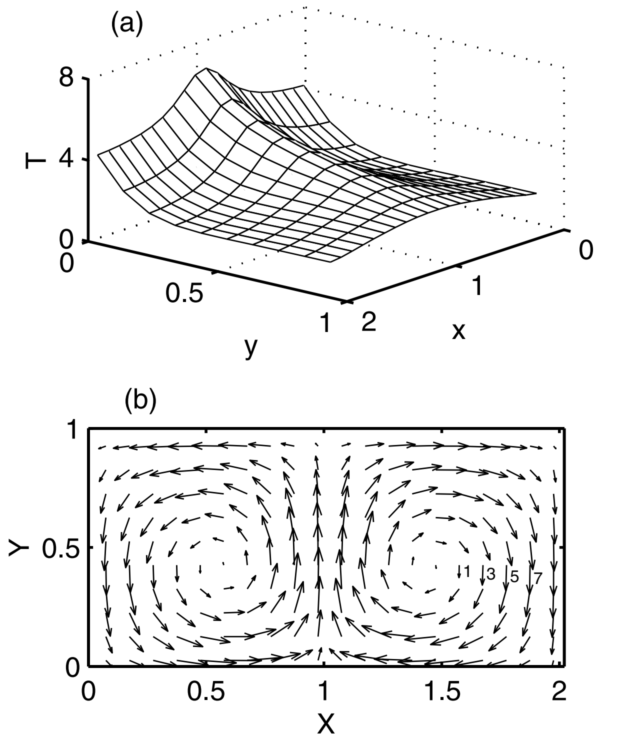

Fig. 2. (a) The temperature surface, (b) the convection roll, for $P = 0.02$.

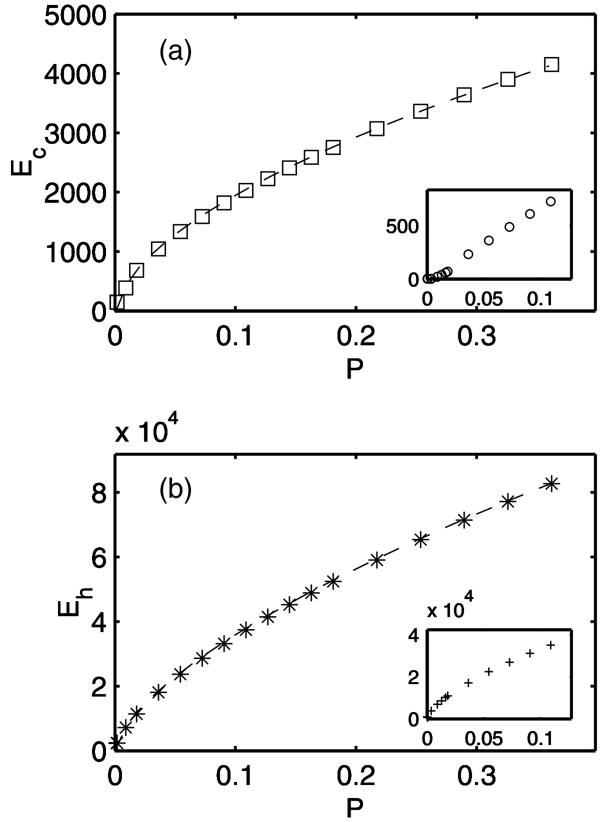

Fig. 3. (a) The convection energy of the system as a function of input power. The dashed line is $E_c = 7.5 \times 10^3 (P - 0.02)^{0.58}$. (b) The heat energy. The dashed line is $E_h = 1.6 \times 10^5 P^{0.65}$. The inset is the corresponding energy for the case with both ν and χ being constant.

appears the temperature is no longer uniform in x direction. This can be explained as follows. In Figure 2(b), two rolls are roughly symmetric about the vertical line bisecting the box. In the layer adjacent to the side wall of the box, falling grains are warmed up. When they reach the middle part of the bottom, their temperature reaches a maximum value. Then they move up along the bisection line due to the buoyancy force. Upon reaching the top surface, they continue to move from the center towards the side wall. Meanwhile they are cooled down.

Convection always develops through a supercritical bifurcation. Figure 3(a) shows the convection energy of the system, which can be fitted with $E_c = 7.5 \times 10^3 (P - 0.02)^{0.58}$. Figure 3(b) shows the kinetic energy of the random motion (or heat energy) of the system, $E_h = \sum \frac{1}{2} v^2$, summing over all finite-difference grid cells, which can be fitted with $E_h = 1.6 \times 10^5 P^{0.65}$. We can see that the ratio of convective energy to heat energy is about 5%, the same order of magnitude as in experiment [9]. This shows that most of the injected energy is transferred to random (heat) energy.

There have been many studies of convection in normal fluids. To show the more active role that ν and χ play here than in normal fluids, we calculate the equations (1–3) with ν and χ being constant (normal fluid), but other conditions remaining the same as in a granular system. The result shows that the convection energy of the system can be fitted with $7.2 \times 10^3 (P - 0.01)$, while the heat energy is approximately the same as that of the granular system (For comparison these two bifurcation diagrams are inserted in corresponding diagrams of Fig. 3, respectively). It is well known that both ν and χ are the factors against the convection. In vibrofluidized granular systems, as P is very small, both ν and χ are smaller than that of a normal fluid, favorable to the occurrence of convection. As P (and then v) is increased, both ν and χ are increased, and they become increasingly unfavorable for convection to occur. In normal fluid these coefficients are independent of v, P must reach some critical value, buoyancy can overcome viscosity, and convection can occur.

To eliminate completely the effect of lateral walls on the onset of convection, we use periodic lateral boundary conditions, and fix the conditions at the bottom and top as in the above calculation. Convection is still observed, sometimes with very slow motion along the lateral direction, even though there is zero total horizontal initial momentum, and vertical boundary conditions do not affect the horizontal moment of the system. Figure 4 is an example of the convection pattern under periodic lateral condition. The periodic structure in the horizontal direction can be observed clearly.

The angular velocity, Ω, is a typical characteristic of the convection roll. We roughly calculate the mean angular velocity of the convection roll by

$$\Omega = \frac{\oint_L \mathbf{u} \cdot d\mathbf{l}}{2S}. \tag{10}$$

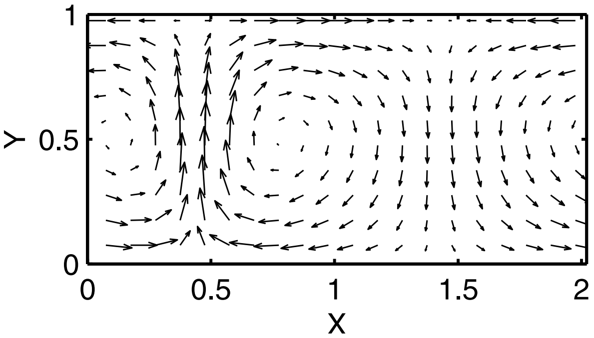

Fig. 4. The convection pattern for periodic lateral boundary condition and for $P = 0.16$.

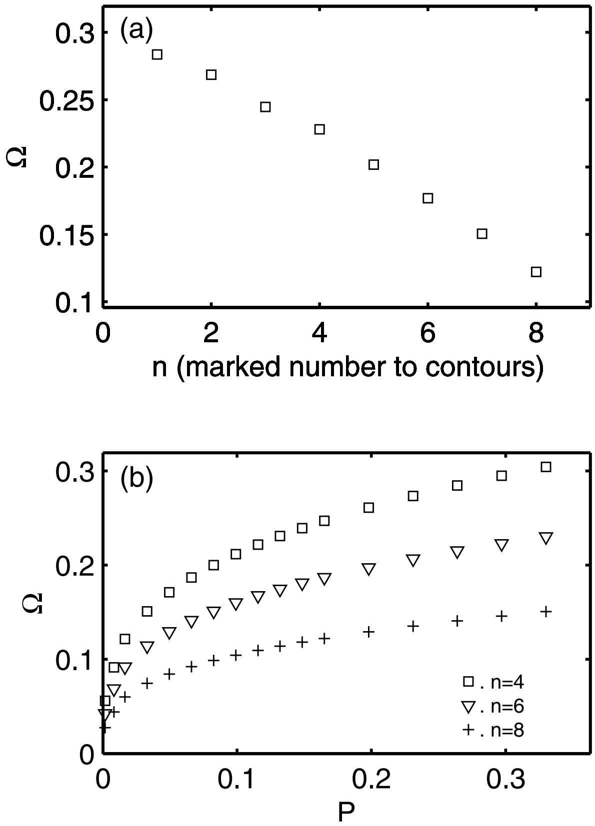

Fig. 5. (a) The distribution of angular velocity from the interior to the exterior of a convection roll for $P = 0.2$. The contours are numbered by 1, 2,. . . , n,. . . , from interior to exterior of the convection roll [as shown in Fig. 2b]. (b) The angular velocity as a function of input power P for three concentric contours numbered 4, 6, 8, respectively.

where L is some closed contour centered at the center of the convection roll, and S is the region encircled by L. We perform the line integral by taking the sum $\sum \mathbf{u} \cdot \triangle\mathbf{l}$ along the contour L. In this way we calculate Ω for several concentric contours and for several different values of P. The results are shown in Figure 5. Figure 5(a) shows that Ω decreases from the interior to the exterior of the roll. We call this roll an active roll, as if some momentum is acting along its axis, resulting in the formation of the gradient in Ω. Similar phenomena have been observed numerically in normal fluids as well as experimentally in both normal fluids and vibrofluidized granular systems. We conclude that active rolls are universal. Figure 5(b) shows the variation of Ω as P is increased for three concentric closed contours. We can see that Ω increases more slowly in outer than in inner layer as P is increased. This is in qualitative agreement with experiment [10].

In summary, the granular-hydrodynamics we present reveal the properties of thermal convection in vibrofluidized granular systems. We observe some universal phenomena of convection in both fluid and vibrofluidized granular system. The granular-hydrodynamics provides a proper approach to systematic investigation of related phenomena in granular systems.

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